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B. E. (Third Semester) Examination, April-May 2021

(New Scheme)

(IT Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *Question (a) of each unit is compulsory & carrying 2 marks. Solve any other **two** questions from each unit & carrying 7 marks.*

Unit-I

1. (a) If $f(x) = x \cdot \sin x$ in $(-\pi, +\pi)$, then the value of

   $b_x = $ ....

   (b) If

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$$

prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$$

(c) Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0, \pi)$. Hence show that

$$\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots \infty = \frac{\pi - 2}{4}$$

(d) Obtain the constant term and the co-efficients of the first sine and cosine terms in the fourier expansion of y as given in the following table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Unit-II

2. (a) State convolution theorem to find inverse Laplace Transform.

(b) Find Laplace transform of :

(i) $t^2 \sin at$

(ii) $\dfrac{1 - e^t}{t}$

(c) Apply convolution theorem to evaluate

$$L^{-1} \frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

(d) Solve :

$$ty'' + 2y' + ty = \sin t \text{ when } y(0) = 1$$

Unit-III

3. (a) The value of

$$\int_C \frac{3z + 4}{z(2z + 1)} dz$$

where C is the circle $|z| = 1$ is ....

(b) If $f(z)$ is analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|Rf(z)\right|^2=2\left|f'(z)\right|^2$$

(c) Find the Laurent's expansion of

$$f(z)=\frac{7z-2}{(z+1)z(z-2)}$$

in the region $1<z+1<3$.

(d) Apply the calculas of residues to find the value of

$$\int_0^{2\pi}\frac{\cos 2\theta}{5+4-\cos\theta}=\frac{\pi}{6}$$

Unit-IV

4. (a) Obtain the partial differential equations by eliminating the arbitrary functions

$$Z=f\left(x^2-y^2\right)$$

(b) Solve :

$$\left(z^2-2yz-y^2\right)p+(xy+zx)q=xy-zx$$

(c) Solve :

$$\left(D^2+2DD'+D'^2-2D-2D'\right)z=\sin(x+2y)$$

(d) Solve by the method of separation of variables :

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$$

Unit-V

5. (a) If the mean of poisson distribution is 'm', then S. D. of this distribution is ....

(b) Four coins are tossed. What is the expectation of number of heads?

(c) The probability that an entering student will graduate is 0·4. Determine the probability that out of 5 students (a) none (b) one and (c) at least one will graduate.

(d) Fit a poisson distrubution to the following data :

x	0	1	2	3	4
f	46	38	22	9	1